MATHEMATICS.
[GEOMETRY.]
Tuesday
1 July 1947.
9.30—12.

UNIVERSITIES OF MANCHESTER, LIVERPOOL, LEEDS, SHEFFIELD AND BIRMINGHAM.

SCHOOL CERTIFICATE EXAMINATION.

TWO AND A HALF HOURS.

Answer ALL questions in Section A and any FOUR questions from Section B.

In answer to Questions 2 (b) and 3 (a) full details and a complete proof are required.

In answer to Questions 1, 2 (a), 3 (b) and 4 no proofs are required, but in calculations sufficient steps in the working must be shown to make clear how the calculation has been performed.

SECTION A.

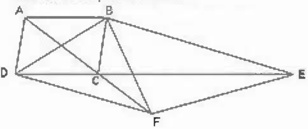
A 1. Construct (both in one figure, not separately) the two triangles ABC, which have $AB=3$ in., $BC=2$ in. and the angle $CAB=30°$.

Construct the perpendicular from B to AC and, making any measurements you wish to make in the figure, calculate the difference between the areas of the two triangles.

A 2. In the accompanying figure $ABCD$ is a parallelogram, F is on AC produced, E is on DC produced, and BE is parallel to DF.

(*a*) Giving your reasons in full, name a triangle in this figure equal in area to the triangle BEF, and also two triangles in the figure each equal in area to the triangle BCD.

(*b*) Prove that the triangle BEF is equal in area to the trapezium $ABEC$.



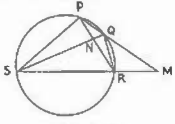
A 3. (*a*) Two straight lines ABC and XYZ are such that AX, BY and CZ are parallel lines. If $AB = BC$, prove that $XY = YZ$.

(*b*) ABD is a semicircle, AD being the diameter. C is a point on the semicircle between B and D, and the tangent at B meets AC produced at E. If the angles BAC and CAD are 20° and 28° respectively, calculate the angles ABC and BEC.

A 4. In the figure (not drawn to scale) name a triangle similar to the triangle QMS, and also a triangle similar to the triangle PQN.

Write down a ratio of sides which equals $QM : QS$, and another ratio which equals $PQ : PN$.

If QM is three-quarters of PM, and RM is two-thirds of QM, and PQ is x inches, calculate the length RS in terms of x.



SECTION B.

Answer any FOUR questions from Section B.

B 5. A quadrilateral $ABCD$ has AB parallel to DC, and the angle A equal to the angle C. Prove that $ABCD$ is a parallelogram.

A five-sided figure $ABCDE$ has AB parallel to ED, and the angles A, B and D are equal and obtuse. Prove that $AE = BC + CD$.

B 6. [Throughout this question neither set square nor protractor are to be used.]

Construct an angle ABC of 90°. Within this angle construct a point P which is 1 in. from BC, and 2·5 in. from B.

Construct the circle which touches both AB and BC, and has its centre as near as possible to P. Measure the radius of this circle.

B 7. State **two** different properties of a quadrilateral, each of which is sufficient to establish that the quadrilateral is cyclic.

Two altitudes AD and BE of a triangle ABC intersect at H. AD is produced to G making $DG = HD$, and ED, CG are produced to meet at K. CH is joined.

Prove that the angles HED and HCD are equal.

Prove also that a circle can be described to pass through B, E, C and K.

B 8. Prove that the internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.

The internal bisector AD of the angle A of the triangle ABC meets BC at D. AB is produced to X, making $BX = BD$ and AC is produced to Y making $CY = CD$. AD produced meets XY at K. Prove that $BD : DC = XK : KY$.

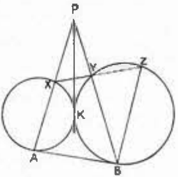
[Turn over.

B 9. Prove that the sum of the squares on two sides of a triangle is equal to twice the square on the median which bisects the third side, together with twice the square on half the third side.

Two points A and B in a plane are six inches apart. A point P moves in the plane so that $PA^2 + PB^2 = 68$ sq. in. Describe as completely as you can the locus of P, giving your reasons.

If, in addition, it is given that the angle APB is to be 40°, state, with reasons, how many possible positions there are for P.

B 10.



In the figure the two circles touch at K and PK is the common tangent at K. AB is another common tangent touching the circles at A, B.

Prove that (a) the points X, Y, B, A are concyclic, (b) BZ is parallel to PA.

[If you can prove (b) by assuming (a), you will get credit for doing so, even if you cannot prove (a).]